FINAL REPORT

DISCLAIMER

TABLE OF CONTENTS

EXECUTIVE SUMMARY

1. This note relates to one basic, and potentially important, point: that physically equivalent constraints may have very different impacts on the regional price.

2. Since the regional price has very significant commercial implications, the choice of constraint representation is a significant issue and a procedure is required to determine an approach to constraint representation that is consistent with both the physical network situation and the commercial structure implied by the NEM rules.

3. The Australian NEM rule is that regional prices should represent the marginal cost of supply to the regional reference node. We show that, in order to produce such prices, a constraint representation must be chosen which has the correct "orientation" with respect to the regional reference node(s). This does not necessarily correspond to the simplest, or most obvious form of the constraint, but can be achieved by using the regional balance equations to substitute for any terms involving regional reference node variables. Alternative pricing rules would require alternative representations.

4. This concept is explained and illustrated, inasmuch as it applies to intra-regional constraints, starting with a simple flow limit that divides a region into two zones, and progressing to a constrained loop structure. These examples suggest the pricing implications of improper orientation are indeed significant, and that correctly oriented constraints can be produced if, but only if:

   - The original constraint is represented in such a way as to include recognition of actual or hypothetical load/generation at all nodes in the region, including the regional reference node, and

   - That constraint representation is then processed, using the regional energy balance equation to substitute for the regional reference node term.

5. The broader question of inter-regional constraint representation is then addressed. Our analysis is somewhat tentative, but it suggests that, for any constraint arising from a line flow limit, the above process can be applied to each region, so as to produce a representation involving only intra-regional generation and interconnector flow terms. This seems intuitive, since such representations properly recognise both the underlying physical situation, and the regional structure of the NEM.

6. It is suggested that the same process also applies to more general constraints, but that hypothesis remains untested.

1. INTRODUCTION

In a fully nodal market model, the transmission system within each region is explicitly modelled, with variables representing the flow through each line, or transmission system component[1]. In such a model it is easy to represent constraints involving limits on flows in particular lines, or on combinations of flows[2]. With care, such models may also be approximated, by only representing flows on key lines, provided the electrical relationships can be modelled with reasonable accuracy.

In principle, it would be possible to introduce critical flow variables into a regional model, too, although to be effective, one would have to also include all variables which affect flow on a particular line. Thus, in the limit, such a "regional" model may be no different from a "nodal" model. But such variables are not permitted in NEMDE, which means that intra-regional transmission constraints need to be expressed solely in terms of generation/load variables.

Although a wide variety of such constraints may be possible, and give rise to pricing effects of significant complexity, we will discuss only simple constraints in which a flow limit is imposed on a particular line. But it turns out that even such simple constraints can be represented in a variety of ways, and that physically equivalent constraints may have very different impacts on the regional price. We start by discussing an important sub-class in which the region is partitioned into two zones, joined by a constrained line (set), so that the line flow limit implies a simple bound on transfers from one zone to the other. We then generalise to more complex cases.

The key factor turns out to be the "orientation" of a constraint with respect to the regional reference node(s), a concept that we introduce and illustrate. We also develop a general approach to transform any constraint, expressed in whatever form may seem natural in the context from which it arose, into a correctly oriented constraint which, when used within the NEMDE, will create regional reference prices which comply with the Code requirement that they should represent the marginal cost[3] of meeting load at the regional reference node.

---

[1] In a network with loops, it is also necessary to introduce "phase angle" variables, so as to allow a DC loadflow calculation.

[2] Phase angle limits can also be applied.

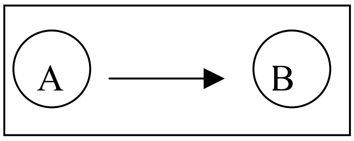
[3] Strictly "marginal value" in the Code.

It should be noted that this concept is, so far as we are aware, new to the international literature, probably because it applies uniquely to a regional market with prices defined as in the Australian NEM. Thus our analysis should be seen as preliminary, and is obviously subject to critique. Further, the analysis, so far, focuses on constraints arising from underlying line flow limits. The theory developed here can certainly be applied to re-orient other constraint forms, too. And it seems likely that this will be appropriate, and probably necessary, if regional reference prices are to be consistently determined. But the implications of performing, or not performing, such re-orientations have not actually been examined, to date.

2. REPRESENTATION OPTIONS FOR SIMPLE LINKING CONSTRAINTS

Consider the situation shown in figure 1, in which two "zones", A and B, exist within a single market region 1. The capacity of the line connecting A and B is FMAX. There may be one or more generators within each zone. We will discuss various ways in which the generation variables may be used to represent the intra-regional transfer limit for this two-zone case, and later look at the pricing implications.

Figure 1: Inter-zonal Transfer Limit Within a Region



Let the total generation in zones A and B be denoted by G_A and G_B respectively. The total demand for the region is denoted by L. There are three alternative ways of representing the intra-regional transmission capacity constraint (FMAX), namely:

1. By constraining off net generation (generation-load) in A such that the export flow to B never exceeds the flow limit, *FMAX*, i.e.:

$$G_A - D_A < FMAX$$

2. By constraining on net generation in B such that the import flow from A never exceeds the flow limit *FMAX, i.e.*

$$G_B - D_B > - FMAX$$

3. By jointly constraining the generation from A *and* B, as discussed below.

In any particular case, there may seem to be an obvious "natural" representation of the constraint, but it is not immediately obvious that there is any general rule for constructing constraints in a consistent fashion, with the choice likely to depend on the relative size of the zones involved, and the ease of measuring particular load components, for example. Thus if zone A contains only a small amount of generation, and no load, then the first constraint representation may seem the obvious choice. Conversely, if zone B contains a major load, and relatively little generation, then the second constraint representation may seem the obvious choice. But we will show that these, seemingly natural, representations are not only inconsistent, but do not necessarily give the "correct" prices.

Note, too, that it is not practical to adopt a constraint representation for which the components are not known in real time. Thus, for example, the latter proposal is impractical unless the load in region B can be monitored in real time. Such a situation was faced in the initial implementation of the Queensland market, and can be resolved as follows.

Let us suppose, for example, that the total load in the region (including losses) is actually determined from the total metered generations in the previous period, G^0_A and G^0_B, as:

$$L = G^0_A + G^0_B$$

Further, it could be assumed that the zonal loads, L_A and L_B, are split in the proportions $w : (1 - w)$, so that the zonal loads for A and B in any period are estimated as:

$$L_A = w \,.\, [G^0_A + G^0_B]$$

$$L_B = (1 - w) \,.\, [G^0_A + G^0_B]$$

But the regional energy balance constraint for any period also requires that the sum of the zonal generation terms equal the total regional load [$L = G_A + G_B$]. Thus, it can also be substituted into, say, a constraint expressed as a limit on the maximum net injection in zone A (that is, representation 1, above), yielding:

$$(1 - w) \,.\, G_A - w \,.\, G_B < FMAX$$

This provides the third representation referred to above. Thus, if we let c_A and c_B be the (single segment) generation offers for generators in zones A and B respectively, the generation dispatch problem for the region would be expressed as:

$$\text{Minimize } c_A \,.\, G_A + c_B \,.\, G_B$$

Subject to:

$$G_A + G_B = L$$

$$(1 - w) \,.\, G_A - w \,.\, G_B < FMAX$$

3. PRICING IMPLICATIONS

Note that all three of the intra-regional constraint representations discussed in the previous section will produce a feasible solution and, given the same data, should produce exactly the same primal (that is, physical) dispatch solution. But they do not produce the same prices.

The shadow price P_d of the regional demand constraint gives the market-clearing price for the region. It is not difficult to show that:

- If the constraint representation constrains generation off in zone A, then the model will produce a "market price" corresponding to the marginal cost of increasing generation in zone B or, equivalently, to the marginal cost of meeting a load increment in zone B;
- If the constraint representation constrains generation on in zone B, then the model will produce a "market price" corresponding to the marginal cost of increasing generation in zone A or, equivalently, to the marginal cost of meeting a load increment in zone A; and
- If the constraint representation discussed above is adopted, then the price will be a weighted average of the marginal costs of increasing generation/meeting load in the two zones where the weights are the proportion of load involved, so that:

$$\text{Regional Price } (P_d) = c_A \,.\, w + c_B \,.\, (1-w)$$

If for example, the marginal cost of generator A (c_A) is \$10/MWh, and that of generator B (c_B) is \$50/MWh, and $w=0.3$, the regional reference price under the three, physically equivalent, constraint representations can be summarised as shown in Table 1.

Table 1: Summary of Intra-regional Constraint Pricing Effects

			Price (P_d) (\$/MWh)
1	Constrain off Generator A	Marginal cost in the importing zone (c_B)	50
2	Constrain on Generator B	Marginal cost in the exporting zone (c_A)	10
3	Jointly constrain Generators A and B	Weighted average of the two marginal costs	38

It should be noted that this hypothetical example has practical ramifications, and that the potential commercial, social, and political implications of these differing regional price estimates could be significant. So which is the "correct price"?

In principle the Australian National Electricity Code answers this question definitively by defining a "Regional Reference Node" for each region, and defining the regional price to be the marginal cost of meeting a load increment at that node[4]. Thus, to be compatible with the Code, the constraint representation must be chosen so as to produce that result, irrespective of any other considerations. Where possible, these reference nodes are chosen to be major load centres.

Such reference nodes will normally also lie in zones that are net importers, like region B in the above example. Thus, the appropriate constraint representation will normally be to constrain distant generation off, rather than constraining local generation on. More generally, though, the philosophy clearly seems to be that, since the market is deemed to exist at the reference node, transactions occurring at that node should be treated as unconstrained. Conversely, then, constraints must be expressed in forms that only constrain variables relating to generation/load away from that node, irrespective of or whether they are being constrained on, or off.

In this simple example, it is relatively easy to choose the constraint representation which corresponds to the NEM Code. We explore the generalisation of this conclusion in the next section, which considers the problem of transforming an inappropriate constraint representation into an appropriate representation. We will not pursue further discussion of the alternative representations. But it is worth observing that, apart from any practical implementation difficulties.

- Under a nodal pricing regime (load weighted) prices at load nodes must, on average, be higher than (generation weighted) prices at generation node. This occurs both because of marginal loss effects, which are accounted for in setting NEM prices, but also because of intra-regional constraint effects, which are not. Thus the NEM practice of defining the regional reference node at a major load centre seems to imply that, typically, the price for any market region will be set to higher than the average price which generation would receive from a nodal pricing regime. Conversely, the reverse practice of setting a reference node in a major generation centre would change the situation, but not obviously improve it, by setting a price which is typically lower than the prices a nodal market would set for load.

---

[4] Pedantically, the Code actually refers to 'marginal value", but this has been interpreted to mean "marginal cost".

- Conversely, while the weighted average constraint representation may seem non-intuitive, the weighted average price it produces most closely approximates the average of the prices which might be produced by the creation of more pricing regions or, in the limit, by a nodal pricing regime. Thus it is, at least arguably, the most appropriate price for the average load, corresponding to the marginal cost of meeting a balanced increment in load across the region, rather than an increment concentrated solely at the regional reference node.

4. ORIENTING CONSTRAINTS IN AN ACYCLIC NETWORK

The process of intra-regional constraint formation described above for two zones may be extended to multiple zones, provided the underlying network connecting the zones forms a tree structure, i.e. it is "acyclic" or "unlooped". One of the zones will be defined as the reference zone, at which "the market" is deemed to exist. Each of the lines joining zones will have a transfer limit in each direction or, equivalently, a conventional direction, with both an upper (positive) and a lower (negative) bound on flows in that direction. It is not necessarily obvious, a priori, which of these bounds will be relevant in expressing a particular constraint, but note that an upper bound on the flow from a zone towards the reference zone would imply "constraining off" generation in that zone, and in zones which supply to "the market" through that zone, while a lower bound in the same direction would imply "constraining on" generation in that zone, and in zones which are supplied from "the market" through that zone.

Thus the key to determining how such flow bounds are to be expressed lies in utilising the tree structure to determine, and constrain, those regions whose access to/from the market are constrained by the flow bound, as follows. Let *n* be any zone, and *next* be its adjacent zone along the unique[5] path from *n* to the reference zone. Let *A(n)* be the set containing *n* and all zones beyond, in the sense that the unique path from each of these zones to the market passes through *n*. Then the bounds on flows from (to) *n* towards (away from) the market reference zone may be expressed as:

$$\sum_{a\in A(n)} (G_a - L_a) \le FMAX_{n,next}$$

$$\sum_{a\in A(n)} (G_a - L_a) \ge -FMAX_{n,next}$$

Applying these formulae to each intra-regional flow bound, in turn, generates an equivalent set of constraints, involving no explicit reference to intra-regional transmission flows. Including these in the market model will ensure a dispatch which is feasible with respect to all such bounds, and a price corresponding to the marginal cost of meeting load at the defined market reference node. Many of these constraints will probably not be binding in practice, though, and the constraint set might be considerably reduced for implementation purposes.

Rather than produce examples for this special case, we proceed to derive, and illustrate, a more general approach.

---

[5] Uniqueness is guaranteed by the tree structure.

5. ORIENTING CONSTRAINTS IN A GENERAL REGIONAL NETWORK

Unfortunately, the above formulae cannot be applied if the zones form loops within the regional network because there is then no unique path from any one zone to the market reference node. In a nodal model, the power flow constraints relating to flows on the lines forming a loop may lead to "spring washer" pricing impacts, of the type discussed in the CRA report on constraint representation.[6]. Such effects cannot be directly assessed unless the intra-regional flows and line characteristics are explicitly modelled, as they would be in a full network model. But such constraints need to be expressed, even in a regional model, and we must ask how such constraints are to be expressed so as to produce regional reference prices corresponding to the marginal cost of supplying load at a reference node.

We should also consider the possibility that there might be intra-regional constraints relating to other concerns such as system security, or stability. These constraints, too, will impact on regional prices, and we must ask how they are to be expressed in order to produce the "correct" prices.

We contend that the answer lies in further generalisation of the principle developed above, namely that the constraint must be expressed with an appropriate orientation toward the market reference node, and that this orientation can be achieved by substituting alternative expressions for the variables occurring in whatever constraint representation may be first derived, in such a way as to have the effect of leaving generation/load at the market reference node unconstrained.

This involves a slight re-interpretation of the process described above. Suppose, in the original example, that a constraint had originally been derived so as to constrain net generation on in zone B sufficiently to keep flows from A to B below some limit. But this representation is now deemed to be inappropriate because the market reference node is in zone B. Thus the constraint must be transformed so as to constrain only those variables not in zone B. This can be achieved by noting that the regional energy balance constraint can be re-arranged to form an expression for the net generation in zone B, i.e.:

$$G_B + G_A = L_A + L_B \Rightarrow G_B - L_B = L_A - G_A$$

Substituting this into the original constraint representation yields essentially the same constraint in the required form, i.e.:

$$G_B > L_B - FMAX \Rightarrow L_A - G_A < FMAX$$

---

[6] *Network Constraint Formulation: Impact on Market Efficiency* CRA Report to NEMMCo, January 2003.

Similarly, in the acyclic network discussed above, if a flow bound had been expressed in terms of the net generation in zones on the market side of the constraint, then that constraint could be transformed into the appropriate form by a similar substitution, again recognising that the regional energy balance constraint can be re-arranged to yield:

$$\sum_{a \in A(n)} (G_a - L_a) = - \sum_{b \notin A(n)} (G_b - L_b)$$

Similar transformations can be applied to a general constraint of the form:

$$\sum_{i \in I} \alpha_i \times (G_i - L_i) < \text{Constant}$$

In particular, we can ensure that the reference node variables do not appear in the final form of the constraint by substituting the following expression derived from the energy balance constraint for any appearance of those variables[7]:

$$(G_{RRN} - L_{RRN}) = - \sum_{i \neq RRN} (G_i - L_i)$$

Thus, if the initial equation was in the above form, the re-oriented constraint representation would be:

$$\sum_{i \neq RRN} (\alpha_i - \alpha_{RRN}) \times (G_i - L_i) < \text{Constant}$$

This transformation needs to be interpreted with some care, though.

It may be tempting to conclude that any traditional constraint representation that does not include any reference node variables is automatically in the appropriate form. But this is not necessarily the case. But, if the constraint relates to generation, say, the traditional representation may only involve references to nodes where generation is known to occur, and may not include any reference node term at all. This does not mean that the constraint is in the appropriate form, though. The effort must be taken to establish the true general form of the constraint, including notional net injection terms for the reference node, and all other nodes[8]. Only then is it in an appropriate form for application of the required transformation.

---

[7] Note that the energy balance constraint yields an expression for the net injection at the reference node. This only works if the generation and load variables there are to be treated symmetrically. But this is sufficient because, as noted elsewhere, the market-clearing price for generation and load would have to be different if generation and load were not treated symmetrically.

[8] A constraint which truly did relate to generation alone would imply a pricing asymmetry between generation and load. But traditional constraint forms used for system operation purposes may ignore nodally specific load terms, or treat them as constant, simply because they are unknown and/or uncontrollable in real time.

The transformation can then be applied to this generalised equation and, depending on α_{RRN}, may involve substantially altered coefficients at the nodes which had traditionally been involved, and new coefficients appearing for nodes which had not traditionally been thought of as being involved in the constraint. For example, a flow bound in the tree structured network described above might traditionally have been represented using terms for nodes, *Gen*, at which generation is dispatched, with the relevant load terms assumed to take an aggregate value, *Localload*, for a specified time of day, say. Thus the constraint may have been expressed in the form:

$$\sum_{\substack{i\in Gen\\ i\notin A(n)}} G_i > Localload\text{ - }X$$

But if we know this constraint is supposed to be a representation of a flow bound on some line, we know that its true form must be:

$$\sum_{a\notin A(n)} (G_a - L_a) > FMAX$$

Note that, since the reference node is not included in *A(n)*, injection there will appear in this generalised constraint, with a coefficient of *+1*, just like any other node on the same side of the flow bound. Thus, substituting the above expression for reference node injection into this equation will subtract *1* from the coefficient for every other node in the network. As a result, nodes outside of *A(n),* which originally had a coefficient of *+1*, will now have a coefficient of 0, while nodes in *A(n)* which originally had a coefficient *0*, will now have a coefficient of *–1*, yielding a new constraint representation:

$$-\sum_{a\in A(n)} (G_a - L_a) \geq -FMAX_{n,next} \quad => \quad \sum_{a\in A(n)} (G_a - L_a) \leq FMAX_{n,next}$$

In this case it is intuitively clear that the result is a valid representation of the original constraint, in the required form, and we have already concluded as much above. But note that, in terms of the coefficients applied to particular terms, it is radically different.

In other cases, the final constraint representation may not bear any obvious resemblance to the original, or have any obvious natural interpretation. Without knowing what a constraint represents, or how it was derived, it will not necessarily be obvious that it is "correctly oriented". But we assert that this is the only representation of this constraint which will produce valid prices, defined in terms of the marginal cost of meeting load at the reference node.[9]

---

[9] That is, it is the only constraint which will directly constrain flows on this line. But, like any other LP constraint, it can be scaled without affecting the solution, or the regional reference price.

An appendix discusses a small example, by reference to standard power flow analysis. But there is a simpler, and perhaps more intuitive, way of understanding this concept.

Figure 2: Illustrative Network

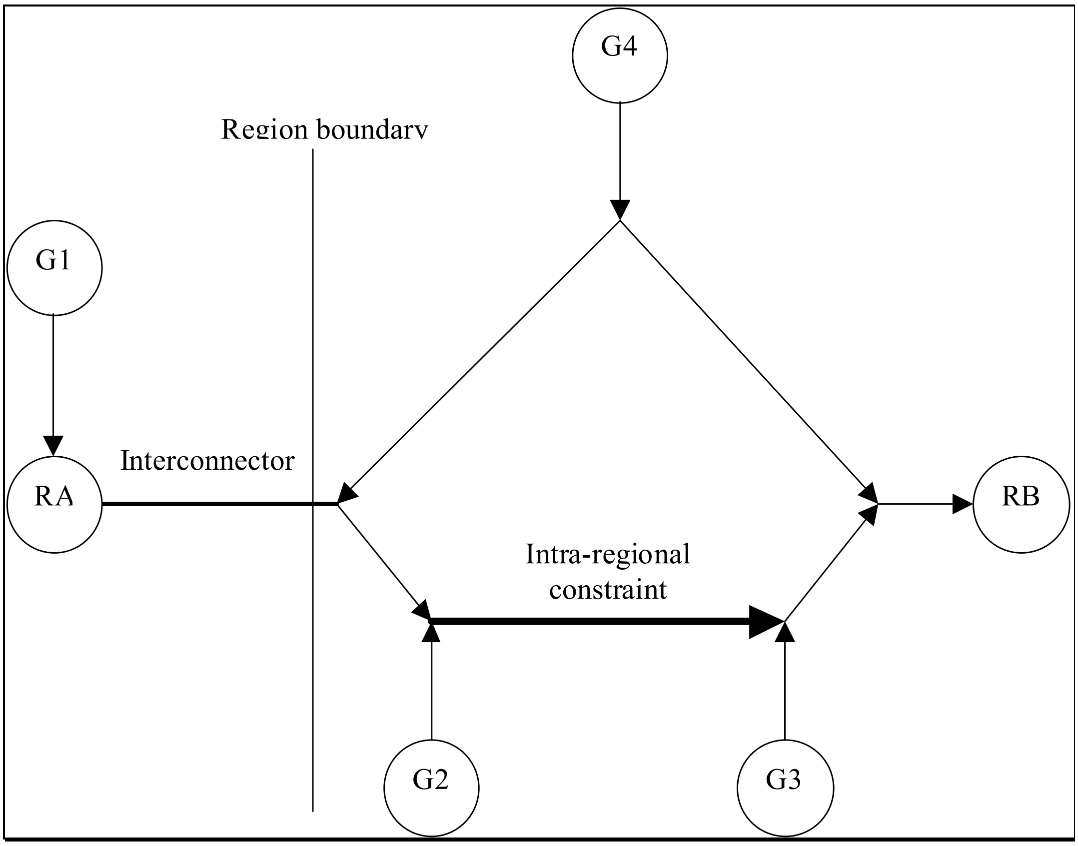


Figure 2 shows the illustrative network discussed in the CRA report. As in that report, we assume that the interconnector (effectively G1) connects to the loop halfway between G2 and G4. We also assume that each of the line segments in the modified network diagram has the same impedance[10]. Thus we can say that the impedance is 1 for each of G2-G1, G1-G4, G4-RB, RB-G3, G3-G2, making a total impedance of 5 around the loop. As in the CRA report, we will assume there is a constraint on flows from G2 to G3. Initially, though, we will ignore the regional boundary, and the possibility of load at RA, and treat G1 as if it were in region B[11].

---

[10] Here "impedance" is loosely equivalent to "resistance", and may be thought of as a measure of "electrical distance"

[11] Noting that a generator does not need to connect directly to a loop in order to influence flow, but it is the point at which flows from that generator enter the loop which is relevant.

The impedances are critical to understanding pricing, and constraint representation, in a looped network, because they determine how power will flow between any two points in the network. There are, in fact, many electrically equivalent ways of deriving and expressing any constraint, corresponding to alternative nodes to which we could imagine incremental power flowing. But, in order to derive constraints in a form which is correctly oriented with respect to the regional reference node, we must consider how an incremental MW would flow from each generator to the regional reference node, to meet an incremental MW of load there.[12].

It is well known that, in such a loop, the incremental flow will split, with the portion travelling around each side of the loop being inversely proportional to the impedance between each generator and the regional reference node around that side of the loop. Thus for G3 and G4, which lie (electrically) close to the regional reference node, a high proportion of any increment will flow directly to the regional reference node, but there will also be some flow around the other side of the loop. But for G2, or G1, which are connected to the loop more or less diametrically opposite the regional reference node, the two flows will be more evenly split. Specifically:

- 0.8 of the flow from G4 will flow directly to the regional reference node, but 0.2 will flow in the opposite direction, and hence contribute to increasing flows over the constrained line;
- 0.6 of the flow from G1 will flow around the unconstrained side of the loop to the regional reference node, but 0.4 will flow on the other side, and hence contribute to increasing flows over the constrained line;
- 0.4 of the flow from G2 will flow around the unconstrained side of the loop to the regional reference node, but 0.6 will flow on the other side, and hence contribute to increasing flows over the constrained line; and
- 0.8 of the flow from G3 will flow directly to the regional reference node, but 0.2 will flow in the opposite direction, and hence contribute to decreasing flows over the constrained line.

Since these same observations apply to all flow, they imply that the constraint on flows from G2 to G3 can be expressed as[13]:

$$0.4G_1 + 0.6G_2 - 0.2G_3 + 0.2G_4 \quad < LIM_{2>3}$$

---

[12] Ignoring losses, obviously

[13] Remember we are, so far, treating G1 as if it were in region B, and ignoring any load at RA.

This is not the only possible constraint representation, but it is the only one correctly oriented to the regional reference node[14]. If, for example, we wanted to make G4 the regional reference node, similar logic suggests that we could express the constraint as[15]:

$$0.2G_1 + 0.4G_2 - 0.4G_3 - 0.2\,(-L_B) < LIM_{\,2>3}$$

It may seem strange that two such different constraints can be equally valid, but recall that it is not the impact of this constraint alone which matters within the NEMDE, but the way in which it interacts with other constraints in the LP, particularly, in this case, with the energy balance constraint which, again ignoring RA, reads:

$$L_B = G_1 + G_2 + G_3 + G_4$$

It is easy to verify that substituting this expression for L_B into the second form of the equation yields the first, as argued above[16]. And it is worthwhile noting that, just as the first representation contained no reference to its reference node (RB), so the second representation contains no reference to its reference node (G4). We repeat the caution, though, that this is not a definitive test, because the relevant term may have been omitted in the original representation of the equation simply because there was no generation/load at the regional reference node, rather than because it is correctly oriented.

---

[14] Although this equation can be scaled without impacting orientation, and this may useful, particularly if we want the right units on the RHS, for pricing purposes.

[15] The double negative applied to the load term implies that it is being treated as "negative generation", and that generation at that position of the network would decrease flows over the constrained line. Naturally, load at that point increases such flows.

[16] Actually, the form of the energy balance constraint is such that a correctly oriented constraint can be achieved by simply adding the coefficient for generation at the reference node (-0.2) to all coefficients in the second constraint form, thus eliminating the regional reference node term, and setting terms which were zero to minus that coefficient (i.e. +0.2 in this case). This would seem to be a general rule.

Finally, we note that, unlike the earlier analysis, this last example ignored the possibility of load terms appearing for nodes other than the regional reference node. Clearly, for constraints of this type, load has the same effect as negative generation, and it may be argued that it should really appear in the same way as generation, as a variable on the LHS of the LP constraint. This is only true, though, if load at each node is actually dispatchable and/or faces nodal prices which cause it to respond as if it were. If load is neither dispatchable nor price responsive, it becomes a constant, in the sense that it is determined by factors outside of the LP optimisation. Thus, while it is true that a general approach to constraint orientation involves load terms being treated in the same way as generation terms, on the LHS of any constraint equation, the final representation will involve transferring those terms to the RHS, where they will be treated as constant. This does not mean that local load terms can be ignored when orienting constraints. Rather, it seems likely that this two stage process will actually have to be adopted, since otherwise the RHS for the re-oriented constraint will not be obvious.

6. ORIENTING CONSTRAINTS WITH MULTIPLE REGIONS

Finally, we must consider the real world case, in which there may be several regions with regional reference nodes, and prices, defined in the market model. Such models may contain separate inter- and intra-regional constraints, as in the NEMDE formulation currently employed in Australia.

More generally, it would appear that the natural form of the constraints limiting flows between regions is really “trans-regional”, with both inter- and intra-regional terms involved in the same constraint. Conversely, many constraints which are “intra-regional’ in the sense that they involve flow limits on intra-regional lines, must also be “trans-regional”, at least in the sense that they involve both (intra-regional) generation terms and (inter-regional) flow terms, since the flow on the critical lines will often be determined by both factors in combination.

The issue arises, then, as to which regional reference node such constraints should be oriented towards:

- If the constraint relates to a flow limit within a region it seems natural to orient the constraint toward that regional reference node. But is that correct? And what happens when we try to include a flow variable in the substitutions described above?

- If the constraint relates to a flow limit between regions, or contains terms involving several inter-regional flows, it is not at all clear what the correctly oriented representation might be.

Our analysis of this issue is, as yet, somewhat tentative. But initial investigation suggests that the answer is that the constraint representation should be oriented toward all of the regional reference nodes involved, or more exactly that substitutions should be made for all of the regional reference node variables. This is possible because each region defined in the formulation has its own energy balance equation, so that there is a one-to-one matching between the number of reference node variables to be eliminated from the formulation, and the number of energy balance equations that can be used to achieve this elimination.

But those energy balance constraints each involve inter-regional flow variables, so that eliminating regional reference nodes from constraints “introduces” inter-regional flow terms…. a not unexpected consequence, given that many of the constraints with which we are concerned relate naturally to interconnector flow limits. So we come to the tentative, but intuitive, conclusion that correct orientation of inter-regional, or trans-regional, constraints can be achieved by expressing them in ways which involve explicit, rather than implicit interconnector terms and, of course, by orienting constraints correctly within each region.

In exploring this hypothesis we have examined some simple examples, which may prove instructive.

First, we can extend the first example to include four nodes, ABCD, forming a linear network, but grouped into two regions, AB and CD, with the flow between them being $F_{AB>CD}$. Suppose that the reference nodes for the two regions are A and D respectively, and that there is a constraint on flows from A to B in region AB, and a constraint on flows from C to D in region CD. Then analysis of the dual (that is, the pricing solution) suggests that:

- The formulation of this problem yields the correct prices, such that the price in each region equals the marginal cost of supplying its reference node, if, but only if, a substitution is made, in each region separately, to eliminate the reference node variables from the expression describing the constrained flow in that region;
- With these substitutions, it is also possible to express trans-regional constraints involving any combination of these two intra-regional flows jointly with each other, or with the inter-regional flow, and still get the correct reference prices in both regions. In terms of the dual analysis, this occurs because, with these substitutions, the (possibly notional) demand variable for the reference node only appears in the regional energy balance equation, so that the regional reference price, being the shadow price on that balance equation, must equate to the marginal cost of meeting that demand, without any adjustment terms such as might arise from involvement of that variable in other equations; and
- Substituting for the regional reference node terms always produces constraint forms involving $F_{AB>CD}$. Conversely, it is possible to use the energy balance equations to derive an expression for $F_{AB>CD}$ in terms of generation variables, and so to produce alternative representations of either constraint, or both. But analysis of the corresponding dual reveals that a formulation with such constraint forms can not yield the correct prices. This relates to the fact that, while it is possible to eliminate either regional reference node in this way, it is impossible to eliminate both simultaneously.

Now consider the example discussed above, reverting to its original form involving two regions. Then it is not really G_1, but $F_{A>B}$, which should appear in the equation, giving:

$$0.4\,F_{A>B} + 0.6G_2 - 0.2G_3 + 0.2G_4 \quad < LIM_{2>3}$$

But then the (very simple) energy balance equation for region A can be re-arranged to give:

$$F_{A>B} = G_1 - D_A$$

So we have:

$$0.4 G_1 - 0.4 D_A + 0.6 G_2 - 0.2 G_3 + 0.2 G_4 \quad < LIM_{2>3}$$

Thus we have two alternative constraint forms. One refers to a regional reference node, but with no flow terms, while the other contains a flow term, and does not refer to a regional reference node.

We might guess that the original representation is “correctly oriented”, not only because it contains no regional reference node term but because the physical constraint is actually entirely within region B, and this form of the constraint contains the representation within region B, using flow terms to represent what are effectively injections into that region. Experiments confirm that it does produce ‘correct’ prices, corresponding to the marginal cost of supplying each regional reference node.

Conversely, the alternative representation gives different prices. In fact, eliminating the flow term in this equation has an important implication. Ignoring the possibility that flows might be constrained by another limit, inter-regional flow actually becomes a redundant concept in the LP. It only appears in the two regional balance equations, and adding those equations together would eliminate the variable entirely, thus also eliminating the regional concept, and leaving only one global energy balance equation. LP experiments suggest that, even if both regional balance equations are retained in the LP, this redundancy becomes evident by the fact that the LP sets the regional reference price in both regions to a single value. Not surprisingly, given the orientation of the constraint to the regional reference node for region B, this single price matches the marginal cost of meeting load in that region[17].

Thus, once more, we come to the, not too surprising, conclusion that a constraint representation that, by eliminating inter-regional flow terms, makes no explicit recognition of the regional boundary, also eliminates that distinction in pricing, and so fails to produce regional reference prices appropriate to the NEM. We also note that, since the most obvious form for “interconnector flow limits” of the kind employed in the NEM would surely involve explicit interconnector flow terms anyway, it should not be difficult to achieve this requirement when orienting constraints.

---

[17] Unless, as noted, another inter-regional flow constraint, expressed using an explicit flow variable, becomes binding.

It should be recognised, though, that these examples are not as general as they might be, inasmuch as they involve situations where the notional interconnector limit is in fact set by an underlying intra-regional constraint. For such problems, it should always be possible to represent the constraint using a combination of intra-regional generation and interconnector flow terms, as in these examples. The real constraint situation is, after all, entirely contained within one region, and the interconnector flow terms simply act as net injections to that region. And each region has its own regional energy balance equation, allowing the elimination of terms involving the regional reference node.

For such constraint structures, nothing more seems to be required. Since the constraint orientation can be achieved entirely within each region, and can be applied to any intra-regional network structure, it seems irrelevant whether the regions themselves are interconnected in an acyclic structure, for example, or in loops. But, leaving aside the issue relating to even more general constraint structures, is it possible that generation terms from several regions would naturally appear in a single constraint and, if so how should we orient it?

First, although effective interconnector capacity certainly can be affected by generation elements on both sides of a regional boundary, it seems reasonable to expect that constraints arising from flow limits on specific lines can always be expressed in terms of a combination of inter-regional flow and intra-regional generation terms, within a single region[18]. Thus situations of this type which can not be expressed without involving cross-border generation terms most likely involve two underlying constraints, one on each side of the boundary, and should be expressed in that way, with each constraint properly oriented within its own region.

Still, such trans-regional constraints could arise for other reasons, driven by other physical and operational considerations. Thus, it is worth considering how they should be handled. We suggest the, as yet untested, hypothesis that neither the inclusion of generation terms from more than one region, nor indeed the inclusion of any other terms, should pose a problem with respect to constraint orientation, provided the representation properly recognises both the underlying physical situation, and the regional structure of the NEM. In other words, provided:

- The original constraint is represented in such a way as to include recognition of actual or hypothetical load/generation at all nodes in each region, including the regional reference nodes; and

---

[18] This includes limits on the interconnector itself, which can be expressed as simple bounds on inter-regional transfer, with no generation terms. Constraints involving loop flows across regional boundaries may be more problematic, if they arise, but initial investigation suggests that they, too, can be expressed in a purely intra-regional form, by netting off the impact which intra-regional generation has on cross border flows when deriving the constraint expression.

- That constraint representation is then processed, using each regional energy balance equation to substitute for each regional reference node term, as above[19].

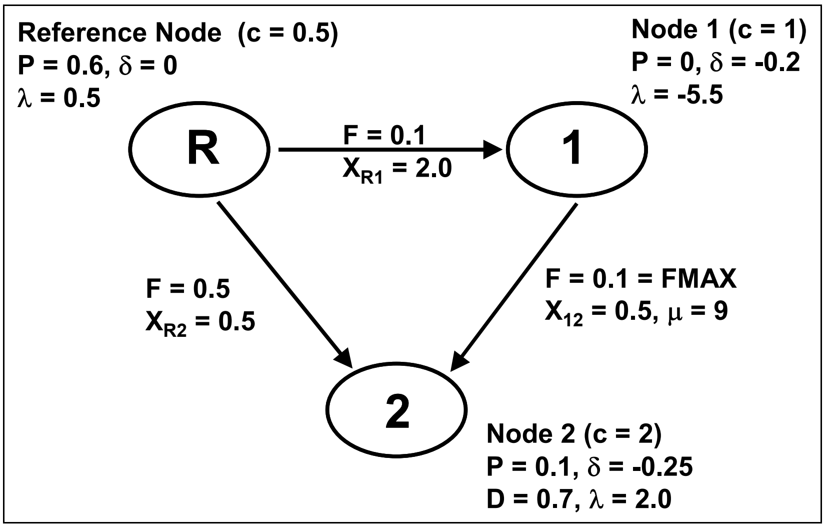
---

[19] Noting that any such substitution must introduce, or impact on, inter-regional flow terms, since they are involved in the energy balance equation.

APPENDIX A: MATHEMATICAL EXAMPLE

This appendix demonstrates the impact of changing constraint orientation using a simple mathematical example[20] wherein the reference node is involved in a loop that comprises three intra-regional links including a constrained one. Figure 3 presents the generation dispatch (***P***) and the associated flows (***F***) to meet a demand (***D***) of 0.7 at node 2, which will be the reference node.

Figure 3: Dispatch and Prices for 3-bus System with Line 1 → 2 flowlimit = 0.1



Generation costs (***c***) for the generators at the three nodes are shown, and we assume there are no generation capacity constraints. Line reactances, X_{ij}, for the three lines are shown, and the flows have been calculated, using standard power flow techniques as being driven by “phase angles”, via[21]:

$$F_{ij}=(\delta_i-\delta_j)/X_{ij}.$$

In order to eliminate the reference node from the flow calculation, we set the phase angle for this node (δ_2) to zero. The optimal dual variables (shadow prices) associated with this dispatch are also shown in Fig.2, namely the nodal prices (λ) corresponding to the demand constraint and congestion prices (μ) for the flow limit on link 1→2.

It is easy to show that the angle variables δ can be eliminated and the constrained flow (F_{12}) can be expressed in terms of the other two flows from the reference node as:

---

[20] Wood and Wollenberg, 1995, 3 bus system for DC load flow, pp. 108

[21] See, for example, Wood and Wollenberg

$$F_{12} = (F_{R2}.X_{R2} – F_{R1}.X_{R1})/ X_{12}$$

We can further eliminate the flows from the reference node by substituting the net generation terms for them from the nodal balances to derive the following expression:

$$F_{12} = [(D_2\text{-}P_2)X_{R2} – (D_1\text{-}P_1)X_{R1}]/ (X_{12} +X_{12} +X_{12}) \leq 0.1$$

Thus, we have now obtained an expression for the constrained flow that does not involve the reference node variables, as desired. We have implemented this form of the flow bound in a regional model and verified that the dispatch, regional (reference node), and congestion prices match those of the nodal model (described in Figure 3). Conversely, we have also verified that the congestion prices produced by retaining the reference node terms in the constraint representation do not match that of the nodal model.